Table of Contents

1	**Course title**	Mathematical Physics 1
2	**Course number**	0302281
3	**Credit hours**	3
	Contact hours (theory, practical)	(3,0)
4	**Prerequisites/corequisites**	General Physics 2 (0302102)
5	**Program title**	B. Sc. In Physics
6	**Program code**	
7	**Awarding institution**	
8	**School**	Science
9	**Department**	Physics
10	**Level of course**	2nd year
11	**Year of study and semester (s)**	2020-2021 – First Semester
12	**Final Qualification**	
13	**Other department (s) involved in teaching the course**	
14	**Language of Instruction**	English
15	**Teaching methodology**	☒Blended ☒Online
16	**Electronic platform(s)**	☒Moodle ☒Microsoft Teams ☐Skype ☐Zoom ☐Others …………
17	**Date of production/revision**	Jan. 17, 2021

18 Course Coordinator:

Name: Nidal M. Ershaidat
Office number: Physics Building - 011
Phone number: 23059
Email: n.ershaidat@ju.edu.jo

19 Other instructors:

Name:
Office number:
Phone number:
Email:

Name:
Office number:
Phone number:
Email:

20 Course Description:

As stated in the approved study plan.
Vector Analysis, Complex Numbers, Linear Algebra, Matrices and Determinants, Ordinary Differential Equations, Fourier Series and Fourier Transform.

21 Course aims and outcomes:

A- Aims:

- Acquire basic mathematical methods, techniques, and skills necessary in physics.
- Solve physics problems through practice.
- Apply knowledge of mathematical techniques in specific realms in physics.
- Use of the space concept. Apply analytical methods (algebra and calculus) in the various encountered spaces (Vectors, Complex space, Matrices and determinants)

B- Intended Learning Outcomes (ILOs):
Upon successful completion of this course, students will be able to:

- Understand the importance of vector analysis in mechanics and electromagnetism.
- Deal with future concepts in quantum mechanics using vector calculus and methods for the solution of differential equations.
- Manipulate simple and relatively advanced calculation in the complex space and the matrices space.
- Solve systems of n equations in n unknowns (linear algebra) using matrices and determinants.
- Solve first and second-order ordinary differential equations
- Calculate the Fourier series (coefficients) for piecewise regular functions.
- Convert coordinates from a coordinate system to another (and vice versa)
- Manipulate vector differential operators in three system of coordinates (Cartesian, cylindrical and spherical)

22. Topic Outline and Schedule:

Week	Lecture	Topic	Teaching Methods*/platform	Evaluation Methods**	References
1	1.1	Overview	Synchronous lecturing/meeting (SL/M) MS TEAMS		
	1.2	Vector Algebra			
	1.3	Vector Multiplication			
2	2.1	Triple Products			
	2.2	Vector Calculus	e-Leareing	Assignment 1	
	2.3	Derivatives of a vector – Velocity, acceleration	(SL/M) MS TEAMS		
3	3.1	Circular Motion			
	3.2	Operators – Scalar and Differential Operators, The Gradient			
	3.3	Applications			
4	4.1	Divergence, curl Significance and use			
	4.2	Laplacian			
	4.3	Problems and Exercises	Moodle	Quiz 1	Attached Documents Q1
5	5.1	Complex Numbers Definition	(SL/M) MS TEAMS		
	5.2	Complex Space Algebra Rectangular and polar forms			
	5.3	Euler's Formula – De Moivre's Formula			
6	6.1	Elementary Functions of a Complex Number			
	6.2	Powes and roots			
	6.3	Trigonometric Functions, exponential, log			
7	7.1	Linear Algebra Matrices			
	7.2	Matrices Space – Algebra and Caculus			
	7.3	Determinants			
8	8.1	Properties of Determinants			

	8.2	Solutions of sets of n equations in n unknowns	(SL/M) MS TEAMS		
	8.3	Special Matrices Complex, orthogonal, and Hermitian Matrices			
				Quiz 2	Document Q2
9	9.1	Curvilinear coordinates	(SL/M) MS TEAMS		
	9.2	Transformation Equations			
	9.3	Conversion of coordinates			
10	10.1	Differential Operators in curvilinear coordinates			
	10.2	Applications			
	10.3		Moodle	Quiz 3	Document Q3
11	11.1	Ordinary Differential Equations (ODEs)	(SL/M) MS TEAMS		
	11.2	Special First-Order ODE			
	11.3	Homogenous 1^{st}-order ODE			
12	12.1	Homogenous 2^{nd} -order ODE			
	12.2	Nonhomogenous 1^{st}-order ODE			
	12.3	Nonhomogenous 2^{nd} -order ODE			
13	13.1	Application in Physics			
	13.2	Fourier Series – Piecewise Regular Functions – Dirichlet's Conditions			
	13.3	Calculating Fourier Coefficients			
14	14.1	Discontinuous Functions			
	14.2	Continuous Functions			
	14.3	Applications			
15	15.1				
	15.2				
	15.3				

- Teaching methods include: Synchronous lecturing/meeting; Asynchronous lecturing/meeting

- Evaluation methods include: Homework, Quiz, Exam, pre-lab quiz…etc

23 Evaluation Methods:

Opportunities to demonstrate achievement of the ILOs are provided through the following assessment methods and requirements:

Evaluation Activity	Mark	Topic(s)	Period (Week)	Platform
Quiz 1	30	Vector Calculus	4	Moodle
Quiz 2	30	Matrices and Determinants	9	Moodle
Quiz 3	30	Curvilinear Coordinates	12	Moodle
Average of Quizzes	**20**			
Mid-Term Exam	**30**	Vector Analysis – Complex Numbers	8 (Nov. 28, 2020)	LMSystem
Final Exam	**50**	All Topics covered	Jan. 12, 2021	JUExams

24 Course Requirements (e.g: students should have a computer, internet connection, webcam, account on a specific software/platform…etc):

Computer, internet connection, the email account @ju.edu.jo

25 Course Policies:

A- Attendance policies: Follow up facilitated by MS TEAMS

B- Absences from exams and submitting assignments on time: Dealt with case by case. Makeup sessions were also held for students whose absence was justified. Some students had technical issues which was the main reason to justify a makeup!

C- Health and safety procedures: Online

D- Honesty policy regarding cheating, plagiarism, misbehavior: According to JU's regulations.

E- Grading policy:

F- Available university services that support achievement in the course: **IT (appreciated)**

26 References:

A- Required book(s), assigned reading and audio-visuals:

Mathematical Methods in the Physical Sciences,
3rd Edition. Mary L. Boas.
Wiley International Edition, 2006.

B- Recommended books, materials and media:

- Mathematical Methods for Physicists, 6th edition
  George B. Arfken and Hans J. Weber,
  Academic Press (Elsevier), 2013.
- *Introduction to Mathematical Physics*,2nd Edition
  Nabil Laham and Nabil Ayoub, 2004,
  مطبعة البهجة – شارع شفيق ارشيدات – إربد

27 Additional information:

See attached appendices

Name of Course Coordinator: Nidal M. Ershaidat Signature: Date: 18/01/2021

Head of Curriculum Committee/Department: --------------------------- Signature: -----------------------

Head of Department: -- Signature: --------------------

Head of Curriculum Committee/Faculty: --------------------------------------- Signature: ----------------

Dean: -- Signature: ---------------------------------------

THE UNIVERSITY OF JORDAN
PHYSICS DEPARTMENT

MATHEMATICAL PHYSICS 1
Phys. 281 (0302281)
FIRST SEMESTER 2020-2021
Prof. NIDAL M. ERSHAIDAT

LECTURE:
SECTION 1 : SUN., TUE., AND THU. 11:30-12:30 (228 ف)
SECTION 2 : MON. AND WED. 11:30-13:00 (228 ف)

TEXT BOOK
Mathematical Methods in the Physical Sciences, 3rd Edition
Author: Mary L. Boas
Publisher: Wiley International Edition, 2006.

USEFUL REFERENCES
All references of the text book +

- Mathematical Methods for Physicists, 6th edition
  George B. Arfken and Hans J. Weber,
  Academic Press (Elsevier), 2013.
- ***Introduction to Mathematical Physics***, 2nd Edition
  Nabil Laham and Nabil Ayoub, 2004,
  مطبعة البهجة – شارع شفيق ارشيدات – إربد

CALENDAR

Week	Title	Chapter (Boas)
1-4	1 Vector Analysis - Algebra 1' Vector Calculus	6
5-6	2 Complex Numbers	2
7-9	3 Linear Equations: Matrices and Determinants	3
10-11	4 Fourier Series	7
12-13	5 Ordinary Differential Equations (ODE's)	8
14	6 Multiple Integrals / Curvilinear Coordinates	5

ASSESSMENT

Exam	Chapters	Date	Marks
Mid-Term	1, 1', 2, 3	Week 8	30%
Assignments- Quizzes			30%
Final	All chapters	To be fixed later	40%

LECTURER INFORMATION

Office	Physics 011
Office Hours	Sun, Mon, Tue and Wed: 10:30-11:30
email	N.Ershaidat@ju.edu.jo
Web sites	http://eacademic.ju.edu.jo/N.Ershaidat
	http://ctaps.yu.edu.jo/physics/Courses/Phys201

MATHEMATICAL PHYSICS 1 Phys. 281 **(0302281)**

Course Description:

This course is intended for the students of the Physics Department at the University of Jordan. It covers important mathematical topics for the understanding of many phenomena in physics.

Course Pre-requisite: General Physics 2 (0301102)

Course Objectives:

- Acquire basic mathematical methods, techniques, and skills necessary in physics.
- Solve physics problems through practice.
- Apply knowledge of mathematical techniques in specific realms in physics.

Steps:

The main objectives of this course are hopefully to be achieved in the following steps:

1. Learning the basic definitions and concepts in the various subjects.
2. Solving a maximum number of problems.
3. Accumulating the knowledge base tightly related to the student's level.
4. Usage of some computational mathematical packages in order to help understanding the concepts.

Chapters Details: (28-29 lectures)

Chapter 1: Vector Algebra: Definition, Properties, Addition, Subtraction, Multiplication of vectors, Rotation of Axes. (4 Lectures)

Vector Calculus: Derivation and Integration, Gradient and Divergence, Gauss's Divergence Theorem, Stokes' Theorem, Potential Theory. (4 Lectures)

Chapter 2: Complex Numbers: Definition and representation of complex numbers, Properties of complex numbers, Functions of a complex variable, Powers and roots of a complex number, Applications. (4 Lectures)

Chapter 3: Matrices & Determinants: Laws and properties of matrices, Special matrices, Matrix inversion, Orthogonal matrices, Eigenvalues and Eigenvectors, Hermitian and Unitary matrix. Properties of determinants, Solution of a set of homogeneous and nonhomogeneous equations. (5 Lectures)

Chapter 4: Ordinary Differential Equations (ODE's): First Order and Second Order Differential Equations. (5 Lectures)

Chapter 5: Fourier Series: Fourier coefficients, General and complex Form of Fourier Series, Properties of Fourier Series. (3 Lectures)

Chapter 6: Curvilinear Coordinates: Orthogonal curvilinear coordinates, Cartesian and spherical coordinates, Cylindrical coordinates, Separation of variables. (3 Lectures)

ملاحظات وتعليمات عامة

1. يُرجى الالتزام بحضور المُحاضرات التي ستُعقد عن بُعد. أنظر تعليمات جلسات المادة عن بُعد.
2. ايقاف استقبال المُكالمات الخلوية حال بدء المحاضرة وحتى نهايتها.
3. فيما يتعلق بالمواظبة والغياب عن المحاضرة يُرجى الاطلاع على المادة (13) من تعليمات منح درجة البكالوريوس في الجامعة الأردنيَّة الصادرة عن مجلس العمداء بقراره رقم (2017/1781) تاريخ 2017/11/27 بموجب الفقرة أ من المادة (3) من نظام منح الدرجات العلميَّة والدرجات الفخريَّة والشهادات في الجامعة الأردنيَّة رقم (58) لسنة 1984.

   أنظر: http://units.ju.edu.jo/ar/LegalAffairs/Regulations.aspx

4. يمكنكم الحصول على خطة المساق ومواد ذات علاقة بالمساق ومتابعة أي إعلانات خاصة به من خلال:
   - التواصل مع المدرس من خلال موقعه على بوابة الجامعة الالكترونيَّة والذي عنوانه http://eacademic.ju.edu.jo/N.Ershaidat
   - الزيارة الدوريَّة للموقع: https://www.dropbox.com/sh/eiahh249pdhgze8/AACA97kEPfz_vu6Xcpdd4ihpa?dl=0
   - ومن خلال الموقع التالي: http://ctaps.yu.edu.jo/physics/Courese/phys201

عنوان المدرس الالكتروني: **n.ershaidat@ju.edu.jo**

1

Mathematical Physics 1 Phys. 281

Physics Department
The University of Jordan,
11942 Amman Jordan

Exceptional Supplement 1

GENERAL RULES

Prof. Nidal M. Ershaidat

2

Microsoft TEAMS®

- Two teams have been created.

UJ - S1 Phys281 - Mathematical Physics 1 0302281

UJ – S2 Phys281 - Mathematical Physics 1 0302281

Lectures

Will be uploaded on OneDrive (accessible using your @ju.edu.jo email)

- PowerPoint Format (Click here)
- Recorded Lectures (Click here)

3

Rules for the TEAMS group

1) This group is strictly reserved for issues-questions related to the course.

2) This group expires one day before the final exam.

3) Do not change the settings of the group!

4) Please! No un-necessary messages.

التهاني وصباح الخير ... إلخ

4

Pre-Session Steps

1) Check your sound devices (and camera).

2) It is recommended to use headphones.

5

Rules – During Audio Sessions

1) For an audio only session, there is nothing special to do.

The best way is to mute your microphone (this allows to avoid noise).

2) When you want to speak to other members, unmute your microphone.

3) Wait for the agreement of the group's manager and always wait few seconds after the last speaker. This avoids interferences!

6

Assessment

Assessment

في ضوء قرارات وزارة التعليم العالي الأخيرة وقرارات مجلسِ عمداء الجامعة الأردنيَّة:

1) سيُعقد امتحانُ منتصفِ الفصل عنْ بُعد (التفاصيل لاحقاً).

1) سيُعقد الامتحانُ النهائي في حرمِ الجامعة (التفاصيل لاحقاً).

2) سيتم تحديدُ مواعيدِ الامتحاناتِ منْ قِبل كليَّة العلوم.

3) سيتخلل هذا الفصل بعض الامتحانات القصيرة (Quizzes)
عن بُعد باستخدام موقع الجامعة elearning.ju.edu.jo

Remarks$_1$

- يُمكنكم مراسلتي لمزيد من المعلومات.
- الرجاء تبادل المعلومات فيما بينكم. يُمكنكم التعاون من أجل حَلِّ بعضِ المشاكلِ الفنيَّة قبلَ اللجوءِ إلي.
- قبلَ اللجوءِ إليّ في أيّ أمرٍ كان، الرجاء مُراعاة أنَّني أتعاملُ في هَذا الفصلِ مع مادتيْن (5 شُعب و132 طالباً.

Remarks$_2$

الرجاء عدم استخدام العاميَّة

STAY TUEND

Quiz 1
(Document Q1)

Due by Sun. Nov. 01, 2020

Quiz 1 - Vector Calculus - Instructions

Quiz 1 will be given Next Thursday November 12, 2020.

1) Login to elearning.ju.edu.jo

There are two methods to access the quiz:

Method 1

a) Open the course's page.
b) Go to topic **CHAPTER 1P - VECTOR ANALYSIS - CALCULUS**
c) Click on Quiz 1 - Vector Calculus

Method 2

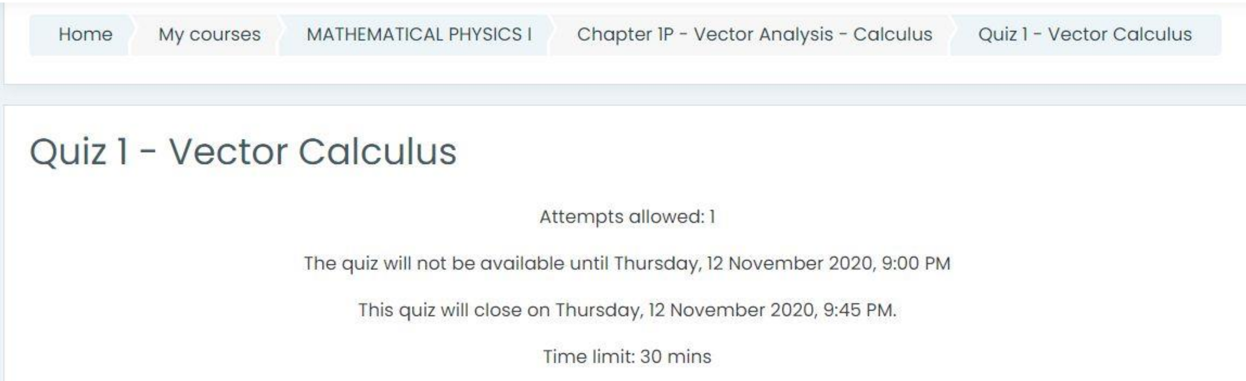
a) click on the following link in the URL bar which redirects you directly to the quiz's page: https://elearning.ju.edu.jo/mod/quiz/view.php?id=306399
b) If this does not work, copy the previous link and paste it in the URL bar of your browser (Chrome is recommended)

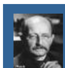
You will be notified that the quiz is closed (See figure below).

It will open on Thursday November 11, 2020 at 09:00 pm and close exactly one hour later, *i.e.* 09:45 pm (same day of course).

You have 30 minutes to answer the questions.

Read the instructions carefully, so you will be ready when you pass it.






PHYSICS DEPARTMENT
THE UNIVERSITY OF JORDAN
MATHEMATICAL PHYSICS 1
PHYS. 281
(0302261)




Quiz 1

Vector Calculus

Gradient

Consider the scalar function

$$\phi(x,y,z) = xy^{\alpha} + \beta y^3 z - \gamma x^2 z^2$$

Let $\vec{X} = \vec{\nabla}\phi = grad\,\phi$

Calculate X_y, *i.e.*, the component y of vector $\vec{X}$ at point P(α,β, 2), *i.e.* $X_y(\alpha,\beta,2)$

Solution

$$X_y = \left[\vec{\nabla}\phi(x,y,z)\right]_y = \frac{\partial\phi}{\partial y} = \alpha x y^{\alpha-1} + 3\beta y^2 z$$

$$X_y(\alpha,\beta,2) = \alpha(\alpha)\beta^{\alpha-1} + 3\beta(\beta)^2(2) = \alpha^2\beta^{\alpha-1} + 6\beta^3$$

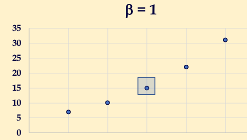
Gradient ✓

$$X_y(\alpha,\beta,2) = \alpha^2\beta^{\alpha-1} + 6\beta^3$$

$$X_y(3,1,\gamma) = (3)^2(1)^2 + 6(1)^3$$

$$= 15$$

β = 1



α	β	γ	X_y	α	β	γ	X_y
1	1		7	1	5		751
2	1		10	2	5		770
3	1		15	3	5		975
4	1		22	4	5		2750
5	1		31	5	5		16375
1	2		49	1	6		1297
2	2		56	2	6		1320
3	2		84	3	6		1620
4	2		176	4	6		4752
5	2		448	5	6		33696
1	3		163	1	7		2059
2	3		174	2	7		2086
3	3		243	3	7		2499
4	3		594	4	7		7546
5	3		2187	5	7		62083
1	4		385	1	8		3073
2	4		400	2	8		3104
3	4		528	3	8		3648
4	4		1408	4	8		11264
5	4		6784	5	8		105472

Divergence ✓

Consider the vector

$$\vec{E}(x,y,z) = \alpha x y^2 \hat{i} + \beta y^3 z \hat{j} - \gamma x^2 z^2 \hat{k}$$

Calculate $\vec{\nabla}\cdot\vec{E}$ at point P(1,-β, 2), *i.e.* $\left(\vec{\nabla}\cdot\vec{E}\right)(P)$

Solution

$$\vec{\nabla}\cdot\vec{E} = \alpha y^2 + 3\beta y^2 z - 2\gamma x^2 z$$

$$\left(\vec{\nabla}\cdot\vec{E}\right)(1,-\beta,2) = \alpha(-\beta)^2 + 3\beta(-\beta)^2(2) - 2\gamma(1)^2(2)^2 = \alpha\beta^2 + 6\beta^3 - 4\gamma$$

Divergence

$$\left(\vec{\nabla}\cdot\vec{E}\right)(1,-\beta,2) = \alpha\beta^2 + 6\beta^3 - 4\gamma$$

$$\left(\vec{\nabla}\cdot\vec{E}\right)(1,-\beta,2) = 4(2)^2 + 6(2)^3 - 4(1) = 60 \rightarrow$$

α	β	γ	Result	α	β	γ	Result
1	1	1	3	1	5	1	771
2	1	1	4	2	5	1	796
3	1	1	5	3	5	1	821
4	1	1	6	4	5	1	846
5	1	1	7	5	5	1	871
1	2	1	48	1	6	1	1328
2	2	1	52	2	6	1	1364
3	2	1	56	3	6	1	1400
4	2	1	60	4	6	1	1436
5	2	1	64	5	6	1	1472
1	3	1	167	1	7	1	2103
2	3	1	176	2	7	1	2152
3	3	1	185	3	7	1	2201
4	3	1	194	4	7	1	2250
5	3	1	203	5	7	1	2299
1	4	1	396	1	8	1	3132
2	4	1	412	2	8	1	3196
3	4	1	428	3	8	1	3260
4	4	1	444	4	8	1	3324
5	4	1	460	5	8	1	3388

Curl ✓

Consider the vector

$$\vec{E}(x,y,z) = \alpha x y^2 \hat{i} + \beta y^3 z \hat{j} - \gamma x^2 z^2 \hat{k}$$

Let $\vec{A} = \vec{\nabla}\times\vec{E} = curl\,\vec{E}$

Calculate A_z, *i.e.*, the component z of vector $\vec{A}$ at point P(α,β, 2), *i.e.* $A_z(\alpha,\beta,2)$

Solution

$$A_z = \left(\vec{\nabla}\times\vec{E}\right)_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \frac{\partial\left(\beta y^3 z\right)}{\partial x} - \frac{\partial\left(\alpha x y^2\right)}{\partial y} = 0 - 2\alpha x y = -2\alpha x y$$

$$A_z(\alpha,\beta,2) = -2\alpha^2\beta$$

Curl

$$A_z(\alpha,\beta,2) = -2\alpha^2\beta$$

$$A_z(4,3,2) = -2(4)^2(3) = -96$$

α	β	A_z	α	β	A_z
1	1	-2	1	5	-10
2	1	-8	2	5	-40
3	1	-18	3	5	-90
4	1	-32	4	5	-160
5	1	-50	5	5	-250
1	2	-4	1	6	-12
2	2	-16	2	6	-48
3	2	-36	3	6	-108
4	2	-64	4	6	-192
5	2	-100	5	6	-300
1	3	-6	1	7	-14
2	3	-24	2	7	-56
3	3	-54	3	7	-126
4	3	-96	4	7	-224
5	3	-150	5	7	-350
1	4	-8	1	8	-16
2	4	-32	2	8	-64
3	4	-72	3	8	-144
4	4	-128	4	8	-256
5	4	-200	5	8	-400

Home | My courses | MATHEMATICAL PHYSICS I | Quizzes - Assessment | Quiz 1 - Vector Calculs | Preview

You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question 1

Not yet answered

Marked out of 10

Curl

Consider the vector

$$\vec{E}(x,y,z) = \alpha x y^2 \hat{i} + \beta y^3 z \hat{j} - \gamma x^2 z^2 \hat{k}$$

Let $\vec{A} = \vec{\nabla} \times \vec{E} = curl\, \vec{E}$

Calculate A_z, *i.e.*, **the component** z **of vector** $\vec{A}$ **at point** $P(\alpha,\beta,2)$, *i.e.* $A_z(\alpha,\beta,2)$

where $\alpha = 7$, $\beta = 3$, and $\gamma = 6$

- -294
- -45
- -276
- -132

Question 2

Not yet answered

Marked out of 10

Divergence

Consider the vector

$$\vec{E}(x,y,z)=\alpha\, x\, y^2\,\hat{i}+\beta\, y^3\, z\,\hat{j}-\gamma\, x^2\, z^2\,\hat{k}$$

Calculate $\vec{\nabla}\cdot\vec{E}$ **at point P(1,-β, 2),** ***i.e.*** $\left(\vec{\nabla}\cdot\vec{E}\right)(P)$

where $\alpha = 2$, $\beta = -2$, and $\gamma = 4$

- -56
- -48
- 40
- 72

Question 3

Not yet answered

Marked out of 10

Gradient

Consider the scalar function

$$\phi(x,y,z) = x\,y^{\alpha} + \beta\,y^{3}\,z - \gamma\,x^{2}\,z^{2}$$

Let $\vec{X} = \vec{\nabla}\phi = grad\,\phi$

Calculate X_y, *i.e.*, **the component** y **of vector** $\vec{X}$ **at point P(α,β, 2),** *i.e.* $X_y(\alpha,\beta,2)$

where α = 3, β = 1, and γ = 6

- ○ 6
- ○ 11
- ○ 15
- ○ 3

Stay in touch

Contact Info

http://www.ju.edu.jo

Mobile : +962 6 5355000

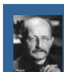
Data retention summary

Get the mobile app

/

Quiz 2
(Document Q2)

Due by Sun. Nov. 01, 2020




PHYSICS DEPARTMENT
THE UNIVERSITY OF JORDAN
MATHEMATICAL PHYSICS 1
PHYS. 281
(0302261)




Quiz 2

Matrices and Determinants

Cofactors

Consider the Matrix ($\alpha \in [1,4]$)

$$\boldsymbol{M} = \begin{pmatrix} \alpha & \beta & -\gamma & 0 \\ 1 & -2 & 3 & 2 \\ 3 & 3 & 2 & 4 \\ 0 & 1 & 7 & 0 \end{pmatrix}$$

Calculate the cofactor c_{34}

Quiz2-1

Cofactors

Solution

$$\boldsymbol{M} = \begin{pmatrix} \alpha & \beta & -\gamma & 0 \\ 1 & -2 & 3 & 2 \\ 3 & 3 & 2 & 4 \\ 0 & 1 & 7 & 0 \end{pmatrix} \qquad \boldsymbol{m}_{34} = \begin{pmatrix} \alpha & \beta & -\gamma \\ 1 & -2 & 3 \\ 0 & 1 & 7 \end{pmatrix}$$

$$\boldsymbol{m}_{34} = -1\begin{vmatrix} \alpha & -\gamma \\ 1 & 3 \end{vmatrix} + 7\begin{vmatrix} \alpha & \beta \\ 1 & -2 \end{vmatrix} = -(3\alpha+\gamma)+7(-2\alpha-\beta) = -17\alpha-7\beta-\gamma$$

$$\boldsymbol{c}_{34} = -\boldsymbol{m}_{34} = 17\alpha+7\beta+\gamma \qquad \boldsymbol{c}_{34} = -\left(\alpha\begin{vmatrix} -2 & 3 \\ 1 & 7 \end{vmatrix} - \beta\begin{vmatrix} 1 & 3 \\ 0 & 7 \end{vmatrix} - \gamma\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}\right) = 17\alpha+7\beta+\gamma$$

Determinant

Consider the Matrix $\boldsymbol{M} = \begin{pmatrix} \alpha & \beta & 0 \\ \gamma & -2 & 2i \\ 3 & 3 & 4 \end{pmatrix}$

Calculate $Re(\boldsymbol{det\ M})$

Solution

$$\boldsymbol{det\ M} = -\left(\alpha \begin{vmatrix} -2 & 2i \\ 3 & 4 \end{vmatrix} - \beta \begin{vmatrix} \gamma & 2i \\ 3 & 4 \end{vmatrix}\right) = \alpha(-8-6i) - \beta(4\gamma - 6i) = \underbrace{(-8\alpha - 4\beta\gamma)}_{Re(\boldsymbol{det\ M})} - (6\alpha + 6\beta)i$$

For $\alpha = 2$, $\beta = -5$, and $\gamma = 4$, we get $Re(\boldsymbol{det\ M}) = 64$

Quiz2-2

System of 3 equation in 3 unknowns

Consider the following set of equations

$$\begin{aligned} \alpha x + 2\beta y - \gamma z &= 10 \\ 2x + y + z &= 9 \\ x - y &= -1 \end{aligned}$$

The value of x is

Quiz2-3

System of 3 equation in 3 unknowns - Solution

$$x = \frac{\boldsymbol{D}_x}{\boldsymbol{D}}$$

$$\begin{aligned} \alpha x + 2\beta y - \gamma z &= 10 \\ 2x + y + z &= 9 \\ x - y &= -1 \end{aligned}$$

We need to calculate the two determinants

$$\boldsymbol{D} = \begin{vmatrix} \alpha & 2\beta & -\gamma \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\gamma \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + (-1) \begin{vmatrix} \alpha & 2\beta \\ 1 & -1 \end{vmatrix} = 3\gamma + \alpha + 2\beta$$

and

$$\boldsymbol{D}_x = \begin{vmatrix} 10 & 2\beta & -\gamma \\ 9 & 1 & 1 \\ -1 & -1 & 0 \end{vmatrix} = -\gamma \begin{vmatrix} 9 & 1 \\ -1 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 10 & 2\beta \\ -1 & -1 \end{vmatrix} = 8\gamma + 10 - 2\beta \Rightarrow x = \frac{\boldsymbol{D}_x}{\boldsymbol{D}} = \frac{8\gamma - 2\beta + 10}{\alpha + 2\beta + 3\gamma}$$

You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question 1

Not yet answered

Marked out of 10

Cofactors

Consider the Matrix

$$M = \begin{pmatrix} \alpha & \beta & -\gamma & 0 \\ 1 & -2 & 3 & 2 \\ 3 & 3 & 2 & 4 \\ 0 & 1 & 7 & 0 \end{pmatrix}$$

Calculate the cofactor c_{34}

Take $\alpha = 9$, $\beta = 7$, and $\gamma = 8$

Answer:

Question 2

Not yet answered

Marked out of 10

Determinants

Consider the Matrix $M = \begin{pmatrix} \alpha & \beta & 0 \\ \gamma & -2 & 2i \\ 3 & 3 & 4 \end{pmatrix}$

Calculate $Re(\mathbf{det}\ M)$

Take α = 4, β = -6, and γ = 11

- 232.0
- 16.0
- -296.0
- 12.0

Question 3

Not yet answered

Marked out of 10

Set of 3 equations in 3 unknowns

Consider the following set of equations

$$\alpha x + 2\beta y - \gamma z = 10$$
$$2x + y + z = 9$$
$$x - y = -1$$

The value of x is

Take α = 9, β = 6, and γ = 7

- 54.000
- 0.778
- 1.286
- 42.000

Quiz 3
(Document Q3)

Due by Sun. Nov. 01, 2020

You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question 1

Not yet answered

Marked out of 10

Spherical to Cartesian

Consider the vector $\vec{E} = \alpha \hat{r}_0 + \beta \hat{\theta} - \gamma \hat{\phi}$
Calculate the component E_y **in Cartesian coordinates at point P defined by** $\theta = \pi$ **and** $\phi = \frac{\pi}{2}$.

Take α = 11, β = 23, and γ = 10

Answer:

Question 2

Not yet answered

Marked out of 10

Cartesian to Cylindrical

Consider the vector $\vec{A} = \alpha\hat{i} + \beta\hat{j} - \gamma\hat{k}$

Calculate the cylindrical component A_r

Take α = 11, β = -3, and γ = 9

- -9.823
- -5.789
- 11.402
- 9.823

Question 3

Not yet answered

Marked out of 10

Divergence

Consider the vector $\vec{E} = \alpha x y \hat{i} + \beta y z \hat{j}$

Let $\Psi(r,\phi,z) = div\,\vec{E}$ **in cylindrical coordinates.**

Calculate $\Psi\left(r=\alpha, \phi=\frac{\pi}{\beta}, z=-1\right)$.

Take α = 9 and β = 2

- 85.000
- 79.000
- 7.000
- 11.000

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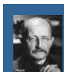


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PHYS. 281
(0302281)




Quiz 3

Curvilinear Coordinates

Cartesian to Cylindrical

Consider the vector $\vec{A} = \alpha\hat{i} + \beta\hat{j} - \gamma\hat{k}$

Calculate the cylindrical component A_r

Quiz3-1

Cartesian to Cylindrical

Solution $\vec{A} = \alpha\left(\hat{r}_0\cos\phi - \hat{\phi}\sin\phi\right) + \beta\left(\hat{r}_0\sin\phi + \hat{\phi}\cos\phi\right) - \gamma\hat{k}$

$$\vec{A} = \hat{r}_0\left(\alpha\cos\phi + \beta\sin\phi\right) + \hat{\phi}\left(-\alpha\sin\phi + \beta\cos\phi\right) - \gamma\hat{k}$$

$$\phi = \tan^{-1}\frac{\beta}{\alpha}$$

$$\hat{i} = \hat{r}_0\cos\phi - \hat{\phi}\sin\phi$$
$$\hat{j} = \hat{r}_0\sin\phi + \hat{\phi}\cos\phi$$

For $\alpha = 2$, $\beta = -5$, and $\gamma = 4$, we get

$$\phi = \tan^{-1}\frac{-5}{2} = -1.1903\ rad \approx -68.2°$$

$$A_\phi = \underbrace{-2\sin(-1.19)}_{1.86} + \underbrace{(-5)\cos(-1.19)}_{-1.86} = 0$$

$$A_r = \underbrace{2\cos(-68.2°)}_{1.22} + \underbrace{(-5)\sin(-68.2°)}_{3.963} \cong 5.383$$

Spherical to Cartesian

Consider the vector $\vec{E} = \alpha \hat{r}_0 + \beta \hat{\theta} - \gamma \hat{\phi}$

Calculate the component E_y in Cartesian coordinates at point P defined by $\theta = \pi$ and $\phi = \dfrac{\pi}{2}$.

Quiz3-2

Spherical to Cartesian

Solution

$$\vec{E} = \alpha \hat{r}_0 + \beta \hat{\theta} - \gamma \hat{\phi}$$

$$\hat{r}_0 = \hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + \hat{k}\cos\theta$$
$$\hat{\theta} = \hat{i}\cos\theta\cos\phi + \hat{j}\cos\theta\sin\phi - \hat{k}\sin\theta$$
$$\hat{\phi} = -\hat{i}\sin\phi + \hat{j}\cos\phi$$

$$\begin{aligned}\vec{E} &= \alpha \hat{r}_0 + \beta \hat{\theta} - \gamma \hat{\phi}\hat{r}_0 \\ &= \alpha\left(\hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + \hat{k}\cos\theta\right) \\ &\quad + \beta\left(\hat{i}\cos\theta\cos\phi + \hat{j}\cos\theta\sin\phi - \hat{k}\sin\theta\right) \\ &\quad - \gamma\left(-\hat{i}\sin\phi + \hat{j}\cos\phi\right)\end{aligned}$$

$$E_x = \alpha\sin\theta\cos\phi + \beta\cos\theta\cos\phi + \gamma\sin\phi$$
$$E_y = \alpha\sin\theta\sin\phi + \beta\cos\theta\sin\phi - \gamma\cos\phi$$
$$E_z = \alpha\cos\theta - \beta\sin\theta$$

Spherical to Cartesian

Solution

$$\underbrace{\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix}}_{S} = \underbrace{\begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix}}_{T} \underbrace{\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}}_{X}$$

$$\underbrace{\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}}_{T^{-1}} \underbrace{\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix}}_{S}$$

Spherical to Cartesian

Solution

$$\underbrace{\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}}_{T^{-1}} \underbrace{\begin{pmatrix} \alpha \\ \beta \\ -\gamma \end{pmatrix}}_{S} \begin{matrix} \leftarrow E_r \\ \leftarrow E_\theta \\ \leftarrow E_\phi \end{matrix}$$

$$= \begin{pmatrix} \alpha\sin\theta\cos\phi + \beta\cos\theta\cos\phi + \gamma\sin\phi \\ \alpha\sin\theta\sin\phi + \beta\cos\theta\sin\phi - \gamma\cos\phi \\ \alpha\cos\theta - \beta\sin\theta \end{pmatrix} = \begin{pmatrix} \alpha\sin\pi\cos\frac{\pi}{2} + \beta\cos\pi\cos\frac{\pi}{2} + \gamma\sin\frac{\pi}{2} \\ \alpha\sin\pi\sin\frac{\pi}{2} + \beta\cos\pi\sin\frac{\pi}{2} - \gamma\cos\frac{\pi}{2} \\ \alpha\cos\pi - \beta\sin\pi \end{pmatrix}$$

$$= \begin{pmatrix} \gamma \\ -\beta \\ -\alpha \end{pmatrix} \qquad E_y = \alpha\sin\theta\sin\phi + \beta\cos\theta\sin\phi - \gamma\cos\phi$$

Divergence

Consider the vector $\vec{E} = \alpha\, x\, y\, \hat{i} + \beta\, y\, z\, \hat{j}$

Let $\Psi(r,\phi,z) = div\,\vec{E}$ in cylindrical coordinates.

Calculate $\Psi\left(r=\alpha, \phi=\frac{\pi}{\beta}, z=-1\right)$.

Quiz3-3

Divergence

Solution

$$\Psi(r,\phi,z) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \alpha\, y + \beta\, z = \alpha\, r \sin\phi + \beta\, z$$

$$\phi = \tan^{-1}\frac{Ey}{Ey} = \tan^{-1}\frac{\beta\, y\overset{0}{} z}{\alpha\, x\, y} = \tan^{-1}\frac{\beta\, z}{\alpha\, r \cos\phi}$$

$$\Psi\left(r=\alpha, \phi=\frac{\pi}{\beta}, -1\right) = \alpha^2 \sin\frac{\pi}{\beta} - \beta$$

For $\alpha = 2$, $\beta = 3$, we have

$$\Psi\left(r=2, \phi=\frac{\pi}{3}, -1\right) = 4\sin\frac{\pi}{3} - 2 \simeq 1.464$$

$$\Psi\left(r=2, \phi=\frac{\pi}{4}, -1\right) = 4\sin\frac{\pi}{4} - 2 \simeq 0.828$$

Mid-Term Exam (Nov. 28, 2020)

Due by Sun. Nov. 01, 2020




PHYSICS DEPARTMENT
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Mid-Term Exam

Nov. 28, 2020

Multiplication of Vectors

Consider the vectors $\vec{A} = 5\hat{k} - 4\hat{j},\ \vec{B} = 2\hat{i} - 3\hat{j}$, **and** $\vec{C} = a\hat{i} - b\hat{k}$,

Let $\vec{D} = \vec{A}\times\left(\vec{B}\cdot\vec{C}\right)$. **If the angle which the vector** $\vec{D}$ **makes with the** ***y-axis*** **is β, then cos β is**

Solution: $\cos\beta = \frac{D_y}{D}$

$\vec{B}\cdot\vec{C} = 2a \Rightarrow \vec{D} = \vec{A}\times\left(\vec{B}\cdot\vec{C}\right) = \left(-4\hat{j}+5\hat{k}\right)(2a) = -8a\hat{j} + 10a\hat{k}$,

We have $|\vec{D}| = \sqrt{164}\,a \Rightarrow \cos\beta = \frac{-8\not{a}}{\sqrt{164}\,\not{a}} = -0.625$ (D_y)

For any values of ***a*** **and** ***b*** **solution is** $\cos\beta \cong -0.625$

Irrotational Vectors

Consider the force $\vec{F} = \left(a x^2 z - b x z\right)\hat{i}$,

(x_c, 1, 1), x_c # 0, is a point where $\vec{F}$ **is** ***conservative.***

Find x_c

Solution: We need to calculate

$\vec{\nabla}\times\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\times\left(a x^2 z - b x z\right)\hat{i} = \left(a x^2 - b x\right)\hat{j}$

$\vec{F}$ **is irrotational if its curl is zero,** ***i.e. if*** $a x^2 - b x = 0$

For $a = -1$ **and** $b = -6$ **solution is** $x_c = \frac{b}{a} = +6$, $\forall\, y$ **and** z.

Divergence

Consider the vector $\vec{E}(x,y,z) = a y^4 z\hat{j} - b x^3 z^2 \hat{k}$

Let $\phi(x,y,z) = \vec{\nabla}\cdot\vec{E}$

Calculate $\phi(1, -b, 2)$.

$\left(\vec{\nabla}\cdot\vec{E}\right)(1,-b,2) = -8ab^3 - 4b$

Solution $\phi(x,y,z) = \vec{\nabla}\cdot\vec{E} = 4 a y^3 z - 2 b x^3 z$

$\left(\vec{\nabla}\cdot\vec{E}\right)(1,-b,2) = 4a(-b)^3(2) - 2b(1)^3(2) = -8ab^3 - 4b$

For $a = -2$, $b = -8$ **solution is** $\left(\vec{\nabla}\cdot\vec{E}\right)(1,-b,2) = -8(-2)(-8)^3 - 4(-8) = -8160$

Laplacian

Consider the scalar function

$$\phi(x,y,z) = a x y^2 + z^b$$

Let $\psi = \nabla^2\phi$, **Calculate ψ at point P(*a*, 4, *b*),** ***i.e.*** **ψ(*a*, 4, *b*)**

Solution

$\psi = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 2ax + b(b-1)z^{b-2}$

$\psi(a,4,b) = 2a(a) + (b)(b-1)(b)^{b-2} = 2a^2 + b^b - b^{b-1}$

For $a = -2$, $b = 5$ **solution is** $\psi(-2,4,5) = \underbrace{2(-2)^2}_{8} + \underbrace{(5)^5}_{3125} - \underbrace{(5)^4}_{625} = 2508$

Multiplication of Complex Numbers

Consider the complex numbers

$$z_1 = 2\alpha + i\beta \text{ and } z_2 = 5 + \gamma i$$

Let $z = z_1 z_2$, calculate $|z| = mod(z)$

Solution: $z = 10\alpha + 2\alpha\gamma i + 5\beta i - \gamma\beta = (10\alpha - \gamma\beta) + (2\alpha\gamma + 5\beta)i$

$$|z| = \sqrt{(10\alpha - \gamma\beta)^2 + (2\alpha\gamma + 5\beta)^2}$$

For $\alpha = 6$, $\beta = 1$, and $\gamma = 2$, we have $|z| = \sqrt{(58)^2 + (29)^2} \cong 64.85$

Division of Complex Numbers

Consider the complex numbers $z_1 = \alpha - i\beta$ and $z_2 = -\gamma i$

Let $z = \dfrac{z_1}{z_2}$

Calculate $Re(z)$

Solution 2: $z = \dfrac{z_1}{z_2}\dfrac{z_2^*}{z_2^*} = \dfrac{z_1 z_2^*}{|z_2|^2} = \dfrac{(\alpha - i\beta)(\gamma i)}{\gamma^2} = \dfrac{\beta\gamma + i\alpha\gamma}{\gamma^2} = \dfrac{\beta}{\gamma} + i\dfrac{\alpha}{\gamma}$

$$Re(z) = \frac{\beta}{\gamma} \quad \text{and} \quad Im(z) = \frac{\alpha}{\gamma}$$

For $\alpha = -2$, $\beta = -6$, and $\gamma = -3$, we have $Re(z) = 2$ and $Im(z) = \dfrac{2}{3}$

Power of a Complex Number

Consider the complex number

$$z = 2\alpha + i\beta$$

Calculate $Re(z^4)$

Solution 2: $r = \sqrt{4\alpha^2 + \beta^2}$ and $\theta = \tan^{-1}\dfrac{\beta}{2\alpha}$

$$z^4 = (r e^{i\theta}) = r^4(\cos 4\theta + i \sin 4\theta) = r^4 \cos 4\theta + i r^4 \sin 4\theta$$

$$\Rightarrow Re(z^4) = r^4 \cos 4\theta$$

For $\alpha = -2$, and $\beta = -3$, we have

$$r = 5 \text{ and } \theta = \tan^{-1}\frac{3}{4} \simeq 217° \Rightarrow Re(z) = 5^4 \underbrace{\cos 4 \times 217°}_{\sim -0.848} = -530$$

Roots of a Complex Number

Consider the complex number $z = 2\alpha + i\beta$

The argument of the second root $\sqrt[4]{z}$ corresponding to $m = 1$, in rad, is

Solution: $r = \sqrt{4\alpha^2 + \beta^2}$, $\theta = \tan^{-1}\dfrac{\beta}{2\alpha} \Rightarrow z = \sqrt{4\alpha^2 + \beta^2}\, e^{i(\theta + 2m\pi)}$

$$m = 1 \Rightarrow \text{second root}: \sqrt[4]{z} = \left(\sqrt{4\alpha^2 + \beta^2}\right)^{1/4} e^{i\left(\frac{\theta}{4} + \frac{\pi}{2}\right)}$$

$$m = 1 \Rightarrow Arg(\text{Second root}) = \frac{\theta}{4} + \frac{\pi}{2} = \frac{1}{4}\tan^{-1}\left(\frac{\beta}{2\alpha}\right) + 1.571$$

For $\alpha = 1$, $\beta = 2$: $r = \sqrt[8]{8}$, $\theta = \dfrac{1}{4}\tan^{-1}\dfrac{2}{2} = \dfrac{\pi}{16} \Rightarrow \theta_2 = \dfrac{\pi}{16} + \dfrac{\pi}{2} = \dfrac{9\pi}{16} \cong 1.77$

Laplacian

Consider the scalar function

$$\phi(x, y, z) = a x y^2 + z^b$$

Let $\psi = \nabla^2 \phi$, Calculate ψ at point P(a, 4, 1), *i.e.* $\psi(a, 4, 2)$

Solution

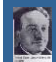
$$\psi = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 2ax + b(b-1)z^{b-2}$$

$$\psi(a, 4, 2) = 2a(a) + (b)(b-1)(2)^{b-2}$$

For $a = -2$, $b = 4$ solution is $\psi(-2, 4, 2) = 2(-2)^2 + (4)(3)(2)^2 = 56$

Final Exam
(Jan. 12, 2021)



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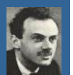

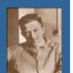

Final Exam
Jan. 12, 2021

3

Potential Theorem

FE-1

Consider the scalar (potential) function

$$\phi(x,y,z) = a\,x\,y^b + z^3$$

Calculate the magnitude of the conservative force associated to ϕ at point P(a,2,b).

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Potential Theorem - Solution

$$\vec{F}(x,y,z) = -\vec{\nabla}\phi(x,y,z) = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(a\,x\,y^b + z^3\right)$$

$$= -\left(\frac{\partial\left(a\,x\,y^b\right)}{\partial x}\hat{i} + \frac{\partial\left(a\,x\,y^b\right)}{\partial y}\hat{j} + \frac{\partial\left(z^3\right)}{\partial z}\hat{k}\right) = -\left(a\,y^b\,\hat{i} + a\,b\,x\,y^{b-1}\,\hat{j} + 3\,z^2\,\hat{k}\right)$$

$$F(x,y,z) = \sqrt{\left(a\,y^b\right)^2 + \left(a\,b\,x\,y^{b-1}\right)^2 + \left(3\,z^2\right)^2}$$

$$F(a,2,b) = \sqrt{\left(a\,2^b\right)^2 + \left(a\,b\,a\,2^{b-1}\right)^2 + \left(3\,b^2\right)^2}$$

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Complex Numbers - Cosine

FE-2

If $z(x) = a\,i\,x$ (where x is real) then $\cos(z(b))$ is:

Solution $\cos(z) = \dfrac{e^{iz}+e^{-iz}}{2} = \dfrac{e^{i(iax)}+e^{-i(iax)}}{2} = \dfrac{e^{ax}+e^{-ax}}{2}$

$$\cos\big(z(b)\big) = \cosh(ab) = \frac{e^{ab}+e^{-ab}}{2}$$

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Inverse Matrix

FE-3

Consider the matrix $A = \begin{pmatrix} \alpha & \beta & 0 \\ \gamma & 2 & 1 \\ 3 & -2 & 0 \end{pmatrix}$

Calculate a_{31}^{-1}

$$AA^{-1} = \frac{1}{2\alpha+3\beta}\begin{pmatrix} \alpha & \beta & 0 \\ \gamma & 2 & 1 \\ 3 & -2 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & \beta \\ 3 & 0 & -\alpha \\ -2\gamma-6 & 2\alpha+3\beta & 2\alpha-\gamma\beta \end{pmatrix}$$

$$= \frac{1}{2\alpha+3\beta}\begin{pmatrix} 2\alpha+3\beta & 0 & \alpha\beta-\alpha\beta \\ 2\gamma+6-2\gamma-6 & 2\alpha+3\beta & \gamma\beta-2\alpha+2\alpha-\gamma\beta \\ 6-6 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Inverse Matrix – Solution

$$\det A = \begin{vmatrix} \alpha & \beta & 0 \\ \gamma & 2 & 1 \\ 3 & -2 & 0 \end{vmatrix} = (-1)\begin{vmatrix} \alpha & \beta \\ 3 & -2 \end{vmatrix} = 2\alpha+3\beta$$

Using Row 2

$$A^{C} = \begin{pmatrix} 2 & 3 & -2\gamma-6 \\ 0 & 0 & 2\alpha+3\beta \\ \beta & -\alpha & 2\alpha-\gamma\beta \end{pmatrix} \Rightarrow A^{CT} = \begin{pmatrix} 2 & 0 & \beta \\ 3 & 0 & -\alpha \\ -2\gamma-6 & 2\alpha+3\beta & 2\alpha-\gamma\beta \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{2\alpha+3\beta}\begin{pmatrix} 2 & 0 & \beta \\ 3 & 0 & -\alpha \\ -2\gamma-6 & 2\alpha+3\beta & 2\alpha-\gamma\beta \end{pmatrix}$$

$$a_{31}^{-1} = \frac{-2\gamma-6}{2\alpha+3\beta}$$

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Inverse Matrix

FE2-3

Consider the matrix $A = \begin{pmatrix} \alpha & \beta & 0 \\ 1 & 2 & \gamma \\ 3 & -2 & 0 \end{pmatrix}$

Calculate a_{31}^{-1}

$$AA^{-1} = \frac{1}{2\alpha+3\beta}\begin{pmatrix} \alpha & \beta & 0 \\ 1 & 2 & \gamma \\ 3 & -2 & 0 \end{pmatrix}\begin{pmatrix} 2\gamma & 0 & \beta\gamma \\ 3\gamma & 0 & -\alpha\gamma \\ -8 & 2\alpha+3\beta & 2\alpha-\beta \end{pmatrix}$$

$$= \frac{1}{2\alpha\gamma+3\gamma\beta}\begin{pmatrix} 2\alpha\gamma+3\beta\gamma & 0 & \alpha\beta\gamma-\alpha\beta\gamma \\ 2\gamma+6\gamma-8\gamma & 2\alpha\gamma+3\beta\gamma & \beta\gamma-2\alpha\gamma+2\alpha\gamma-\beta\gamma \\ 6\gamma+6\gamma & 0 & 2\alpha\gamma+3\gamma\beta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Inverse Matrix – Solution

$$\boldsymbol{det\,A} = \begin{vmatrix} \alpha & \beta & 0 \\ 1 & 2 & \gamma \\ 3 & -2 & 0 \end{vmatrix} = (-\gamma)\begin{vmatrix} \alpha & \beta \\ 3 & -2 \end{vmatrix} = 2\alpha\gamma + 3\beta\gamma$$

Using Col. 3

$$\boldsymbol{A}^{C} = \begin{pmatrix} 2\gamma & 3\gamma & -8 \\ 0 & 0 & 2\alpha+3\beta \\ \beta\gamma & -\alpha\gamma & 2\alpha-\beta \end{pmatrix} \Rightarrow \boldsymbol{A}^{CT} = \begin{pmatrix} 2\gamma & 0 & \beta\gamma \\ 3\gamma & 0 & -\alpha\gamma \\ -8 & 2\alpha+3\beta & 2\alpha-\beta \end{pmatrix}$$

$$\Rightarrow \boldsymbol{A}^{-1} = \frac{1}{2\alpha\gamma+3\beta\gamma}\begin{pmatrix} 2\gamma & 0 & \beta\gamma \\ 3\gamma & 0 & -\alpha\gamma \\ -8 & 2\alpha+3\beta & 2\alpha-\beta \end{pmatrix}$$

$$a_{31}^{-1} = \frac{-8}{2\alpha\gamma+3\beta\gamma}$$

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Adjoint Matrix

FE-4

If $\boldsymbol{M} = \begin{pmatrix} 2 & a-i & i \\ bi & 0 & i \\ b & 0 & -2 \end{pmatrix}$, $\boldsymbol{Q} = \boldsymbol{M} + 2\,\boldsymbol{I}$,

where $\boldsymbol{I}$ is the (3×3) unit matrix.

Calculate $mod\left(\boldsymbol{det\,Q}^{\dagger}\right) = \left|\boldsymbol{det\,Q}^{\dagger}\right|$

and q_{ij} is the element i,j of $\boldsymbol{Q}^{\dagger}$.

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Adjoint Matrix – Solution$_1$

$$\boldsymbol{Q} = \begin{pmatrix} 2 & a-i & i \\ bi & 0 & i \\ b & 0 & -2 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & a-i & i \\ bi & 2 & i \\ b & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{Q}^{*} = \begin{pmatrix} 4 & a+i & -i \\ -bi & 2 & -i \\ b & 0 & 0 \end{pmatrix} \Rightarrow \boldsymbol{Q}^{\dagger} = \begin{pmatrix} 4 & -bi & b \\ a+i & 2 & 0 \\ -i & -i & 0 \end{pmatrix}$$

$Im(q_{12}) = -b$

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Adjoint Matrix – Solution$_2$

$$\boldsymbol{det\,Q}^{\dagger} = \begin{vmatrix} 4 & -bi & b \\ a+i & 2 & 0 \\ -i & -i & 0 \end{vmatrix} = (b)\begin{vmatrix} a+i & 2 \\ -i & -i \end{vmatrix}$$

$$\boldsymbol{Q}^{\dagger} = \begin{pmatrix} 4 & -bi & b \\ a+i & 2 & 0 \\ -i & -i & 0 \end{pmatrix}$$

$$= b(-ai+1+2i) = b + (-ba+2b)i$$

$$\left|\boldsymbol{det\,Q}^{\dagger}\right| = \sqrt{b^2 + b^2(2-a)^2} = b\sqrt{a^2-4a+5}$$

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Adjoint Matrix FE2-4

If $M = \begin{pmatrix} 2 & a-i & i \\ b & 0 & i \\ bi & 0 & -2 \end{pmatrix}$, $Q = M + 2I$,

where I is the (3×3) unit matrix.

Calculate $mod\left(det\, Q^\dagger\right) = \left|det\, Q^\dagger\right|$

and q_{ij} is the element i,j of $Q^\dagger$.

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Adjoint Matrix – Solution$_1$

$$Q = \begin{pmatrix} 4 & a-i & i \\ b & 2 & i \\ bi & 0 & 0 \end{pmatrix}$$

$$Q^* = \begin{pmatrix} 4 & a+i & -i \\ b & 2 & -i \\ -bi & 0 & 0 \end{pmatrix} \Rightarrow Q^\dagger = \begin{pmatrix} 4 & b & -bi \\ a+i & 2 & 0 \\ -i & -i & 0 \end{pmatrix}$$

$Im(q_{13}) = -b$

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Adjoint Matrix – Solution$_2$

$$det\, Q^\dagger = \begin{vmatrix} 4 & b & -bi \\ a+i & 2 & 0 \\ -i & -i & 0 \end{vmatrix} = (-bi)\begin{vmatrix} a+i & 2 \\ -i & -i \end{vmatrix} \quad Q^\dagger = \begin{pmatrix} 4 & b & -bi \\ a+i & 2 & 0 \\ -i & -i & 0 \end{pmatrix}$$

$$= (-bi)(-ai+1+2i) = -ba - bi + 2b = (2b-ab) - bi$$

$$\left|det\, Q^\dagger\right| = \sqrt{b^2(2-a)^2 + (b)^2} = b\sqrt{a^2 - 4a + 5}$$

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System of 3 equation in 3 unknowns FE-5

The following set of equations

$$\begin{aligned} ax + by - 3z &= 0 \\ cx - y + z &= 0 \\ 3x + z &= 0 \end{aligned}$$

has a non-trivial solution if c =

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System of 3 equation in 3 unknowns - Solution

A non-trivial solution exists if ***det A*** = 0

$$\boldsymbol{det\,A} = \begin{vmatrix} a & b & -3 \\ c & -1 & 1 \\ 3 & 0 & 1 \end{vmatrix} = (3)\begin{vmatrix} b & -3 \\ -1 & 1 \end{vmatrix} + (1)\begin{vmatrix} a & b \\ c & -1 \end{vmatrix}$$

$$= (3b-9)+(-a-bc)$$

$$\boldsymbol{det\,A} = 0 \Leftrightarrow (3b-9)+(-a-bc) = 0$$

$$\Leftrightarrow c = \frac{3b-a-9}{b}$$

Using Row 3

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System of 3 equation in 3 unknowns - Solution

A non-trivial solution exists if ***det A*** = 0

$$\boldsymbol{det\,A} = \begin{vmatrix} a & b & -3 \\ c & -1 & 1 \\ 3 & 0 & 1 \end{vmatrix} = (-b)\begin{vmatrix} c & 1 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} a & -3 \\ 3 & 1 \end{vmatrix}$$

$$= (3b-bc)-(a+9)$$

$$\boldsymbol{det\,A} = 0 \Leftrightarrow 3b-bc-a-9 = 0$$

$$\Leftrightarrow c = \frac{3b-a-9}{b}$$

Using Col. 2

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System of 3 equation in 3 unknowns FE2-5

The following set of equations

$$ax + by + 3z = 0$$
$$cx - y + z = 0$$
$$3x - z = 0$$

has a non-trivial solution if c =

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System of 3 equation in 3 unknowns - Solution

A non-trivial solution exists if ***det A*** = 0

$$\boldsymbol{det\,A} = \begin{vmatrix} a & b & +3 \\ c & -1 & 1 \\ 3 & 0 & -1 \end{vmatrix} = (3)\begin{vmatrix} b & 3 \\ -1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} a & b \\ c & -1 \end{vmatrix}$$

$$= (3b+9)-(-a-bc)$$

$$\boldsymbol{det\,A} = 0 \Leftrightarrow 3b+9+a+bc = 0 \Leftrightarrow c = -\left(\frac{3b+a+9}{b}\right)$$

Using Row 3

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Curl in Spherical Coordinates FE-6

Consider the vector $\vec{E} = a\,x\,y\,\hat{i} - b\,z\,\hat{j}$

Let $\vec{X} = \vec{\nabla}\times\vec{E}$. Calculate, $X_r\left(r=2,\theta=\frac{\pi}{3},\phi=\pi\right)$

r, θ, and ϕ are the spherical coordinates.

$\hat{i} = \hat{r}_0 \sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$

$\hat{j} = \hat{r}_0 \sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$

$\hat{k} = \hat{r}_0 \cos\theta - \hat{\theta}\sin\theta$

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Curl in Spherical Coordinates - Solution

$$\vec{E} = \alpha\,x\,y\,\hat{i} - \beta\,z\,\hat{j}$$

$$\vec{X} = \vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \alpha x y & -\beta z & 0 \end{vmatrix} = \beta\hat{i} - \alpha x\hat{k}$$

$$\vec{X} = \beta\hat{i} - \alpha x\hat{k} = \beta\left(\hat{r}_0 \sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi\right) - \alpha r\sin\theta\cos\phi\left(\hat{r}_0\cos\theta - \hat{\theta}\sin\theta\right)$$

$$\vec{X} = \hat{r}_0 \underbrace{\left(\beta\sin\theta\cos\phi - \alpha r\sin\theta\cos\phi\cos\theta\right)}_{X_r} + \hat{\theta}\left(\beta\cos\theta\cos\phi + \alpha r\sin^2\theta\cos\phi\right) - \hat{\phi}\left(\beta\sin\phi\right)$$

$$\Rightarrow X_r(r,\theta,\phi) = (\beta - \alpha r\cos\theta)\sin\theta\cos\phi$$

$$\Rightarrow X_r\left(r=2,\theta=\frac{\pi}{3},\phi=\pi\right) = \left(\beta - \alpha(2)\left(\frac{1}{2}\right)\right)\left(\frac{\sqrt{3}}{2}\right)(-1) = \frac{\sqrt{3}}{2}(\alpha-\beta)$$

FE-7

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Curl in Spherical Coordinates FE2-6

Consider the vector $\vec{E} = a\,x\,y\,\hat{i} - b\,z\,\hat{j}$

Let $\vec{X} = \vec{\nabla}\times\vec{E}$. Calculate, $X_\theta\left(r=2,\theta=\frac{\pi}{3},\phi=\pi\right)$

r, θ, and ϕ are the spherical coordinates.

$\hat{i} = \hat{r}_0 \sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$

$\hat{j} = \hat{r}_0 \sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$

$\hat{k} = \hat{r}_0 \cos\theta - \hat{\theta}\sin\theta$

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Curl in Spherical Coordinates

$$\vec{X} = \hat{r}_0\left(\beta\sin\theta\cos\phi - \alpha r\sin\theta\cos\phi\cos\theta\right) + \hat{\theta}\underbrace{\left(\beta\cos\theta\cos\phi + \alpha r\sin^2\theta\cos\phi\right)}_{X_\theta}$$

$$- \hat{\phi}\left(\beta\sin\phi\right)$$

$$X_\theta(r,\theta,\phi) = \beta\cos\theta\cos\phi + \alpha r\sin^2\theta\cos\phi$$

$$X_\theta(r,\theta,\phi) = \left(\beta\cos\theta + \alpha r\sin^2\theta\right)\cos\phi$$

$$\Rightarrow X_\theta\left(r=2,\theta=\frac{\pi}{3},\phi=\pi\right) = \left(\beta\left(\frac{1}{2}\right) + \alpha(2)\left(\frac{\sqrt{3}}{2}\right)^2\right)(-1) = -\frac{3\alpha+\beta}{2}$$

FE-7

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Homogeneous 2nd-Order ODE

FE2-7

Consider the differential equation: $y'' - a\,y' + 3\,y = 0$

If $y(0) = 0$ and $y'(0) = 3$, then $y(b) =$ $\quad a > \sqrt{12}$

Solution: Auxiliary Equation $D^2 - a\,D + 3 = 0$

$$Roots: \alpha = \frac{a+\sqrt{a^2-12}}{2}, \beta = \frac{a-\sqrt{a^2-12}}{2} \Rightarrow y(x) = A\,e^{\alpha x} + B\,e^{\beta x}$$

$$y(0) = 0 \Rightarrow A = -B \Rightarrow y(x) = A\left(e^{\alpha x} - e^{\beta x}\right) and\ y'(0) = 3 \Rightarrow A = \frac{3}{\alpha-\beta}$$

$$y(x) = \frac{3}{\alpha-\beta}\left(e^{\alpha x} - e^{\beta x}\right) \Rightarrow y(b) = \frac{3}{\alpha-\beta}\left(e^{\alpha b} - e^{\beta b}\right)$$

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Homogeneous 2nd-Order ODE

FE2-7

Consider the differential equation: $y'' - a\,y' + 5\,y = 0$

If $y(0) = 0$ and $y'(0) = 2$, then $y(b) =$ $\quad a > \sqrt{20}$

Solution: Auxiliary Equation $D^2 - a\,D + 3 = 0$

$$Roots: \alpha = \frac{a+\sqrt{a^2-20}}{2}, \beta = \frac{a-\sqrt{a^2-20}}{2} \Rightarrow y(x) = A\,e^{\alpha x} + B\,e^{\beta x}$$

$$y(0) = 0 \Rightarrow A = -B \Rightarrow y(x) = A\left(e^{\alpha x} - e^{\beta x}\right) and\ y'(0) = 3 \Rightarrow A = \frac{2}{\alpha-\beta}$$

$$y(x) = \frac{2}{\alpha-\beta}\left(e^{\alpha x} - e^{\beta x}\right) \Rightarrow y(b) = \frac{2}{\alpha-\beta}\left(e^{\alpha b} - e^{\beta b}\right)$$

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Homogeneous 2nd-Order ODE

FE2-9

Checking $y(x) = \frac{2}{\alpha-\beta}\left(e^{\alpha x} - e^{\beta x}\right)$,

$y'(x) = \frac{2}{\alpha-\beta}\left(\alpha\,e^{\alpha x} - \beta\,e^{\beta x}\right)$, $y''(x) = \frac{2}{\alpha-\beta}\left(\alpha^2\,e^{\alpha x} - \beta^2\,e^{\beta x}\right)$ $\quad a > \sqrt{20}$

$2\left(\alpha^2 e^{\alpha x} - \beta^2 e^{\beta x}\right) - a\,2\left(\alpha\,e^{\alpha x} - \beta\,e^{\beta x}\right) + 3\,2\left(e^{\alpha x} - e^{\beta x}\right)$ [redacted]

$$= \frac{2}{\alpha-\beta}\left[\underbrace{\left(\alpha^2 - a\,\alpha + 3\right)}_{0} e^{\alpha x} - \underbrace{\left(\beta^2 - a\,\beta + 3\right)}_{0} e^{\beta x}\right] \equiv 0$$

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Fourier Series

FE-10

Consider the piecewise function $f(x) = \begin{cases} -A & -1 < x \le 0 \\ A & 0 < x \le 1 \end{cases}$

We define $f_m(x)$ as the sum of the first m terms of the Fourier series.

Calculate $f_7(B)$.

$$f_m(x) = \frac{a_0}{2} + \sum_{n=1}^{m} a_n \cos n\,x + \sum_{n=1}^{m} b_n \sin n\,x$$

Fourier Series - Solution

Consider the piecewise function $f(x)=\begin{cases} -A & -1<x\leq 0 \\ A & 0<x\leq 1 \end{cases}$

$$f(x)=\sum_{n\ odd}^{\infty}\frac{12}{n\pi}\sin n\pi x$$ See Complement A

$$f_7(x)=\frac{12}{\pi}\left(\frac{\sin\pi x}{1}+\frac{\sin 3\pi x}{3}+\frac{\sin 5\pi x}{5}+\frac{\sin 7\pi x}{7}\right)$$

$$f_7(B)=\frac{12}{\pi}\left(\frac{\sin\pi B}{1}+\frac{\sin 3\pi B}{3}+\frac{\sin 5\pi B}{5}+\frac{\sin 7\pi B}{7}\right)$$